Propagation of apertured Bessel beams

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The propagation features of several apertured Bessel beams are numerically calculated. The calculations show that the relations of axial intensity versus propagation distance are similar to the radial distribution of the aperture functions, which may be helpful in choosing the proper aperture functions in experiments.

Key words: Bessel beam, diffraction of light, aperture. © 1995 Optical Society of America

1. Introduction

The $J_0$ Bessel beam is a nondiffracting beam; that is, its transversal distribution does not change as it propagates. Because the $J_0$ beam was first reported and experimentally synthesized by Durnin et al.,\textsuperscript{1,2} there has been much research on its generation,\textsuperscript{3–13} propagation,\textsuperscript{14–17} applications,\textsuperscript{5,9,13,18–21} etc. The ideal apertured $J_0$ beam possesses nearly nondiffracting properties within some axial distance $Z_{\text{max}}$, and its narrow central lobe is practically unchanged within $Z_{\text{max}}$. In the experiment one may come across a nonideal apertured nonhard-truncated $J_0$ beam. On the other hand, although the ideal apertured $J_0$ beam behaves as a nondiffracting beam within $Z_{\text{max}}$, its axial intensity oscillates along the direction of propagation. This behavior is often undesirable in applications. Therefore it is necessary to investigate the properties of propagation of various apertured $J_0$ beams. In this paper we present the numerical results of the propagation of apertured $J_0$ beams, which are modified by several envelope functions (i.e., Gaussian, anti-Gaussian, Airy, triangle). We found an interesting phenomenon: The relationships of the axial intensity versus propagation distance are similar to the radial distributions of the aperture functions. The axicon-generated beam is shown as an example of how to control the propagation of an apertured $J_0$ beam with this phenomenon.

2. Calculations

The $J_0$ beam is circularly symmetric, and our calculations are concerned only with circularly symmetric apertures. Let $(x_1, y_1, z)$ and $(x_2, y_2, z)$ be the coordinates of a pair of points on the incident and diffracted planes, respectively; $\rho$, $\theta$ and $r$, $\phi$ their cylindrical coordinates; and $R$ the distance between them. In the Fresnel approximation the amplitude $A(x_2, y_2, z)$ at a distance $z$ can be obtained from the diffraction integral\textsuperscript{22}

$$A(x_2, y_2, z) = \frac{k}{iz} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x_1, y_1, 0) \exp(ikR) \, dx \, dy,$$

where

$$R = |(x_1 - x_2)^2 + (y_1 - y_2)^2 + z^2|^{1/2}$$

$$= z + \frac{r^2 + \rho^2}{2z} - \frac{2\rho \cos(\theta - \varphi)}{2z}.$$

Note that the incident amplitude $A(x_1, y_1, 0)$ is circularly symmetric and the diffraction field can be obtained from the above equations:

$$A(r, z) = \exp\left(ikz + \frac{ikr^2}{2z}\right)$$

$$\times \int_{0}^{\infty} \rho A(\rho, 0) J_0\left(\frac{kp\rho}{z}\right) \exp\left(\frac{ikp^2}{2z}\right) \, d\rho,$$  \hfill (1)

where $k$ is the wave number, $J_0$ is the zero-order Bessel function, and

$$A(\rho, 0) = F(\rho) J_0(\alpha \rho),$$  \hfill (2)

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where \( F(\rho) \) is the aperture function. If we define \( \xi = \rho/a, \)
\( F(\rho) \) is chosen as
\[
F(\rho) = \begin{cases} 
1 & \text{hard hole,} \\
\exp\left(\frac{\xi^2 - 1}{4}\right) & \text{anti-Gaussian,} \\
\exp(-\xi) & \text{Gaussian,} \\
2J_1(1.22\pi\xi)/(1.22\pi\xi) & \text{Airy,} \\
1 - \xi & \text{triangle,} 
\end{cases}
\]

for \( \rho < a \), whereas \( F(\rho) = 0 \) where \( \rho > a \). \( J_1 \) is the first-order Bessel function. Figure 1 shows a plot of Eq. \((3)\).

When \( r = 0 \) in Eq. \((1)\), we obtain the axial intensity curves:
\[
I(0, z) = |A(0, z)|^2 = \left| k \right|^2 \int_0^a \rho F(\rho) J_0(\alpha \rho) \exp\left(\frac{ik\rho^2}{2z}\right) d\rho.
\]

We choose \( k = 0.9929 \times 10^4 \) mm\(^{-1}\) (the He–Ne laser), \( a = 4 \) mm, and \( \alpha = 40 \) mm\(^{-1}\). Using Eqs. \((3)\) and \((4)\), we calculate the \( I(0, z) \) versus \( z \) curves, which are shown in Fig. 2. Curve a (hard hole) looks very much like the Fresnel diffraction pattern of the straight edge; however, here the curve shows the relationship between the axial intensity \( I(0, z) \) versus the propagation distance. When Fig. 2 is compared with Fig. 1, it is interesting to see that the \( I(0, z) \) versus \( z \) curves are similar to \( A(0, 0) \) versus \( \rho \). In Fig. 2 \( I(0, z) \) is \( \sim 1 \) at small \( z \) for a, c, d, and e because the values of their corresponding aperture functions are nearly 1 for small \( \rho \). \( I(0, z) \) at small \( z \) for the anti-Gaussian function, curve b, is less than 1 because the anti-Gaussian aperture function is less than 1 for small \( \rho \). For \( \rho \approx a \) the values of the hard hole, a, and anti-Gaussian, b, aperture functions are close; the behaviors of their corresponding axial intensities \( I(0, z) \) are similar at the large propagation distance \( z \) (for example, when \( z > 0.9 \) m in Fig. 2).

This tells us that the near-field behavior of \( I(0, z) \) is determined by \( A(\rho, 0) \) of small \( \rho \), whereas the far-field behavior of \( I(0, z) \) is determined by \( A(\rho, 0) \) of \( \rho \approx a \). This is useful because we can to some extent control the \( I(0, z) \) versus \( z \) relationship easily by changing the \( A(\rho, 0) \) versus \( \rho \) distribution. Also, for the Airy and triangle apertures, \( I(0, z) \) does not oscillate; this may be an advantage over the hard aperture because the axial intensity and width of the primary maximum change monotonically and can be easily predicted.

3. Phase Aperture

Nearly diffraction-free beams can be generated by an axicon illuminated by a plane wave, and this method is highly efficient. The axicon-generated beam is used as an example to show how to control \( I(0, z) \) by choosing the proper aperture function \( A(\rho, 0) \).

An axicon is a phase aperture and can be described by phase function \( \exp(-i\alpha \rho) \). Equation \((1)\) gives
\[
I(0, z) = \left| k \right|^2 \int_0^a \rho \exp\left(\frac{ik\rho^2}{2z} - i\alpha \rho\right) d\rho.
\]

Fig. 2. Axial intensities versus propagation distance corresponding to Fig. 1.

Fig. 3. Axial intensities after an axicon versus propagation distance: a, uniform plane-wave incidence; b, Gaussian-beam incidence.
shortly after the axicon is used, we can deduce that the radial distribution of amplitude is more similar to curve b, Fig. 2, at large hard-apertured Bessel beam. Curve a, Fig. 3, is more similar to curve b, Fig. 2, at large z; therefore we can deduce that the radial distribution of amplitude shortly after the axicon is \( J_0 \) multiplied by a function similar to the anti-Gaussian function.

Consequently it is not difficult to improve the axicon-generated beam by a Gaussian aperture or by Gaussian-beam illumination. Then \( I(0, z) \) is obtained by

\[
I(0, z) = \left( \frac{k}{2} \right) \int_0^\infty \exp(-r^2/2\omega^2) \exp\left(\frac{i k p^2}{2 z} - i \alpha p\right) dp.
\]

We calculated Eq. (6) with \( \omega = 1.2 \alpha \) and \( \alpha = 20 \text{ mm}^{-1} \), as shown in Fig. 3, curve b. Curve b, Fig. 3, is more similar to curve a, Fig. 2. Another aperture function might be chosen to give a better result.

On the other hand, instead of the amplitude-aperture function in Eq. (3), phase-aperture functions might be used in the calculations of Eq. (4). For example, let

\[
F(p) = \begin{cases} \exp(-i \alpha p) & \text{axicon}, \\ \exp(i \alpha p) & \text{negative axicon}. \end{cases}
\]

The results of Eq. (4) are shown in Fig. 4. A comparison with curve a, Fig. 2, shows that the negative axicon phase aperture greatly increases the distance of nondiffracting propagation at the price of decreasing the axial intensity, whereas the axicon phase aperture has the opposite effect.

### 4. Conclusion

The influence of the aperture function on the propagation of the \( J_0 \) beams was investigated. Because of the similarity between the axial-intensity variation and the amplitude-aperture function, we can control the central intensity as a function of distance by changing the aperture function. When an Airy or triangle aperture is used, the axial intensity does not oscillate. The phase aperture can also change the central intensity. Therefore this can help control the propagation behavior of the beam.

### References