Conoscopic holography: toward three-dimensional reconstructions of opaque objects

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Conoscopic holography is an interferometric technique that permits the recording of three-dimensional objects. A two-step scheme is presented to recover an opaque object’s shape from its conoscopic hologram, consisting of a reconstruction algorithm to give a first estimate of the shape and an iterative restoration procedure that uses the object’s support information to make the reconstruction more robust. The existence, uniqueness, and stability of the solution, as well as the convergence of the restoration algorithm, are studied. A preliminary experimental result is presented.

Keywords: Image reconstruction, image restoration, computer vision, holography.

1. Introduction

Conoscopic holography is an incoherent monochromatic light Fresnel holographic technique proposed in 1985, with the aim of building a three-dimensional (3-D) camera, i.e., a camera that records both the image and the shape of objects. The use of spatially incoherent light makes it possible to use this technique in various environments, and its resolution is compatible with CCD sensors, which permits the interface with a computer for the digital processing of the holograms. The principle and the basic equations of conoscopic holography are given in Section 2. Then, in Section 3, a 3-D reconstruction algorithm, which is based on rewriting the hologram in a differential form, estimates the shape of an opaque object from its conoscopic hologram. Yet because of the very physics of the interference phenomenon, the reconstruction will not be stable for the low spatial frequencies. In Section 4, in order to recover these low frequencies, we study the properties of an iterative method that takes advantage of the knowledge of the object’s support, important information that to our knowledge has been overlooked. Finally, an encouraging experimental 3-D reconstruction is presented.

2. Principle and Basic Equations

The basic system, called a conoscope because it produces the well-known conoscopic figures, is shown in Fig. 1. A uniaxial crystal C is sandwiched between two circular polarizers P1 and P2. In the on-axis configuration the crystal axis is parallel to the geometrical axis, Oz, of the system.

As with other similar techniques, each object point S produces, on the recording plane, a Gabor zone pattern that encodes both its lateral and longitudinal positions. This pattern results from the interference of two longitudinally displaced points that are the ordinary and the extraordinary images of the original point source through the birefringent crystal. This pattern is the basic point-spread function (PSF) of the system, defined by the equation

\[ R^i(x, y) = \frac{1}{2} \left[ 1 + \cos(\pi f_r x^2 + y^2) \right]; \]  

where \( x \) and \( y \) are the coordinates in the recording plane, and \( f_r \) is the Fresnel parameter, a scale factor that depends on the distance \( z \) between the point and the recording plane:

\[ f_r = \frac{k}{\pi z^2}. \]

In Eq. \( 2 \), \( k \) is a dimensionless constant that depends on the wavelength and the opto-geometrical parameters of the system (the length and the indices of
refraction of the crystal, and \( z_c \) is the so-called conoscopic corrected distance,\(^2\) that is, the geometrical mean distance of the ordinary and the extraordinary images to the recording plane. The difference between the geometrical and the arithmetical mean distance of the ordinary and the extraordinary wave coming from each point source and, as stated previously, neglecting the difference between \( z \) and \( z_c \), we obtain the PSF:

\[
R_c(x, y) = \frac{1}{z^2} \exp \left[ -\frac{ik}{z} (x^2 + y^2) \right].
\]

The conoscopic hologram of an opaque 3-D object is characterized by functions \( I \) and \( z \) reads

\[
H(x', y') = \iint I(x, y)R_c(x, y') \delta(x - x', y - y') dx dy.
\]

Notice that this expression is not the classical Fredholm integral of the first kind, since essentially we seek to recover \( z(x, y) \), which is encoded in the kernel of the integral and mainly in its phase. The algorithm is based on rewriting the integral expression of the hologram in a differential form; the first (and probably easier) derivation of this differential expression was performed in Fourier space.\(^{13}\) The following novel derivation is done directly in real space and advantageously describes the relationship between the reconstruction of the image and that of the shape, which permits a more intuitive understanding of the several solutions, some of them impractical although ingenious, have been suggested.\(^5,9\) In conoscopic holography, there are efficient ways to remove both the bias\(^5,10\) and the conjugate image.\(^11\) These improvements are based on numerical combination of different system PSF’s, each of them being obtained by an adequate change in the input polarization state (with a liquid-crystal light valve) and modulation of the amplitude of the incident light field (with a rotating mask; see Ref. 11). The resulting PSF is shown to be \( R_c \) and gives good quality 2-D reconstructions.\(^12\) In Section 3 the theoretical PSF of Eq. (5) is assumed to derive an algorithm for the reconstruction of the shape of an opaque 3-D object.

3. Three-Dimensional Reconstruction

A. Differential Expression of the Hologram

As mentioned above, the hologram \( H \) of a complete object is the incoherent superposition of the Gabor zone patterns of all the points, but it is no longer a convolution when the object is three dimensional. This is even precisely the feature that enables the recovery of the object’s shape from its hologram. Note that the volume hologram (i.e., the set of 2-D holograms taken in parallel planes) of a true 3-D object is a 3-D convolution, but the 2-D hologram of a nonplanar object cannot be expressed as a convolution because the PSF varies with \( z \). Assuming the object is opaque, all the relevant information about the object can be described as two functions: the image \( I(x, y) \) and the range map \( z(x, y) \) between every point in the object and the recording plane. Taking into account the attenuation of the intensity of the spherical wave coming from each point source and, as stated previously, neglecting the difference between \( z \) and \( z_c \), we obtain the PSF:

\[
R_c(x, y) = \frac{1}{z^2} \exp \left[ -\frac{ik}{z} (x^2 + y^2) \right].
\]

The conoscopic hologram of an opaque 3-D object characterized by functions \( I \) and \( z \) reads

\[
H(x', y') = \iint I(x, y)R_c(x, y') \delta(x - x', y - y') dx dy.
\]
expression. In a 3-D formulation the hologram reads

\[ H(x, y) = \int dz J(x, y, z) *_{2-D} R^z_{c}(x, y), \]

where \( *_{2-D} \) denotes 2-D convolution in the variables \( x \) and \( y \); \( J \) is the 3-D intensity function, which, with an opaque object, reads

\[ J(x, y, z) = I(x, y) \delta(z = z|x, y). \]

This latter expression states that the 3-D line in space going through a given \( (x, y) \) point on the sensor and perpendicular to it intersects the object only once, at \( z = z|x, y \). Let \( z_0 \) be some average distance between the object and the sensor. In order to take into account the fact that the object lies around this mean longitudinal position, Charlot suggested expanding the PSF into a series; along the same line, a first-order development of the PSF \( R^z_{c} \) in the variable \( z^2 \) around \( z_0^2 \) (rather than in the variable \( z \) around \( z_0 \), cf. below) yields

\[ H(x, y) = \int dz J(x, y, z) *_{2-D} R^z_{c}(x, y) + |z^2 - z_0^2| \frac{\partial R^z_{c}}{\partial z^2} \big|_{z = z_0} (x, y); \]

thus

\[ H(x, y) = \left[ \int dz J(x, y, z) *_{2-D} R^z_{c}(x, y) \right] \]

\[ + \left[ \int dz J(x, y, z) |z^2 - z_0^2| \right] \]

\[ *_{2-D} \frac{\partial R^z_{c}}{\partial z^2} |_{z = z_0} (x, y) \]

\[ = I(x, y) *_{2-D} R^z_{c}(x, y) \]

\[ + |I(x, y)| |z^2 - z_0^2| *_{2-D} \left( \frac{\partial R^z_{c}}{\partial z^2} \right) |_{z = z_0} (x, y). \]

A detailed analysis of this first-order approximation is presented in Ref. 15. In particular, let \( \Delta F \) be the longitudinal extension of the object expressed in number of fringes; it can be shown analytically that this error is at worst \( \sim 10\% \) and only in the highest frequencies for \( \Delta F < \pm 0.25 \) fringe, and simulations indicate that reconstructions are still of good quality for \( \Delta F \leq \pm 2 \) fringes, which is perfectly compatible with typical experimental conditions of conoscopic holography for macroscopic and even for microscopic objects.

The order zero of the development corresponds to the object’s image \( I(x, y) \), whereas the first order contains information about the shape \( z(x, y) \). Thus the quantity

\[ R_{3-D} = \frac{\partial R_{c}}{\partial z^2} \]

is termed the 3-D PSF to distinguish it from the 2-D PSF \( R_c \). To avoid notational complexity, we omit the explicit \( z \) dependence of the PSF’s. The 2-D and 3-D transfer functions (TF’s) are, respectively,

\[ \tilde{R}_{c}(\mu, \nu) = \frac{i \pi}{k} \exp \left[ -\frac{i \pi^2}{k} \frac{1}{2} (\mu^2 + \nu^2) \right], \]

\[ \tilde{R}_{3-D}(\mu, \nu) = \frac{\partial \tilde{R}_{c}}{\partial z^2} |_{\mu, \nu} = -\frac{i \pi^2}{k} \mu^2 + \nu^2 \tilde{R}_{c}(\mu, \nu). \]

The 3-D TF is equal to the 2-D TF modulated by a parabola, which corresponds to a Laplacian in the real domain (from this expression, it is already clear that the low frequencies of the shape will not be recorded well, which motivates the restoration procedure presented later in this paper). An inverse Fourier transform (FT) of the 3-D TF yields

\[ R_{3-D}(x, y) = \frac{\partial R_c(x, y)}{\partial z^2} = \frac{i}{4k} \Delta R_{c}(x, y). \]

This can be stated in the following way: the 3-D PSF \( \text{[the one with which } I|x^2 - z_0^2 | \text{ is convolved in Eq. (11)] is the longitudinal derivative of the 2-D PSF \( \text{[with which } I \text{ is convolved.]} \) It equals, within a constant multiplicative factor, the Laplacian of the 2-D PSF. And the fact that this constant is imaginary will enable one to separate the shape from the image in the 3-D reconstruction presented below. Also note that this simple explicit expression of the 3-D PSF was obtained by differentiation of the PSF \( R_c \) with respect to \( z^2 \) (cf. Eqs. 13 and 14), whereas, in Refs. 5 and 14, the differentiation was done with respect to \( 1/z^2 \) and did not lead to an explicit expression of the 3-D PSF.

B. Three-Dimensional Reconstruction Algorithm

In order to separate the two terms in Eq. (11), first we convolve the hologram with the inverse of \( R_c \) of the mean plane \( z_0 \), as if the object were planar. In other words a refocusing of the hologram into the mean plane of the object is performed; we term this the 2-D reconstruction because it yields the image of the object as the real part of the backpropagated, or refocused, hologram:

\[ H'(x, y) = I(x, y) + \frac{i}{4k} \Delta I(x, y)|_{z_0^2} \]

The real part of the 2-D reconstruction is the image of the object, and the imaginary part contains informa-
tion about the shape of the object, in the form of the Laplacian $|\Delta|$ of the product intensity by shape.

At this point it is useful to force to zero the imaginary part of $H'$ when there is no object, i.e., when the real part is close to zero. This will avoid the amplification of artifacts such as an imperfectly removed conjugate image in the 3-D reconstruction process described hereafter. The 3-D reconstruction itself consists essentially of a FT of the imaginary part of $H'$, a multiplication by

$$
\hat{T}_{H}(\mu, \nu) = \frac{-1}{|\mu^2 + \nu^2|}
$$

(17)

and an inverse FT.

Because of the singularity of this filter at the origin and in order to prevent excessive noise magnification, which is typical of ill-posed inverse problems, it is preferable to replace it with a regularized one, for instance, with the Wiener-type filter

$$
\hat{T}_{w}(\mu, \nu) = \frac{-|\mu^2 + \nu^2|}{|\mu^2 + \nu^2|^{2} + w^4},
$$

(18)

where $w$ is a parameter to be adjusted to the noise level in the hologram. This algorithm has been validated by simulation on holograms calculated with $R_{a}$ as the PSF and gives good results. Nevertheless, the relationship Eq. (14) between the 2-D and 3-D TF’s shows that any defect on the 2-D PSF (even if the PSF does give good quality 2-D reconstructions) will be amplified in the low frequencies during the 3-D reconstruction process.

4. Restoration of the Reconstruction

A. Motivation

The main obstacle to 3-D reconstructions of experimental holograms is the difference between the theoretical and the experimental PSF’s. An efficient way to take into account this difference is to incorporate a priori information on the shape to be recovered, i.e., the knowledge of the object’s support, which is given by the 2-D reconstruction (the refocusing step of the 3-D reconstruction). The reconstruction of a 3-D object by the algorithm described in Section 3 is considered in this section as a first approximation of the shape and is used to initialize an iterative restoration method that uses the support constraint.

The use of an iterative scheme for the restoration has several advantages, among which is the absence of the need to implement the inverse of the degradation filter. The support information and the knowledge on the object’s shape given by the reconstruction can both be expressed as projection operations. This formulation takes advantage of a well-established mathematical framework and leads to easily implementable algorithms. Each constraint on the unknown shape forces it to belong to a set of functions that is characterized by its projector if the set is a linear manifold or, more generally, a convex set.

The quantity estimated by the reconstruction algorithm is the product of the image $I$ with the shape itself, i.e.,

$$
f(x, y) = I(x, y)[z(x, y) - z_{0}].
$$

(19)

It is the function that will be restored by means of the support constraint, which simply states that $f$ should be zero when $I$ is zero. In practice, less than a few percent of either the maximum or the average value of $I$:

$$
|S| : f(x, y) = 0, \text{if } I(x, y) = 0
$$

(20)

The reconstruction algorithm theoretically yields all Fourier components of $f$, but it is known that the values of the low frequencies are erroneous because of the divergence at the origin of the theoretical reconstruction transfer function. The knowledge of the object’s support enables one to loosen the Fourier-domain constraint, e.g., to let frequencies $|\mu, \nu|$ in a disk $D_{0}$ loose and to force the sole higher frequencies to their values obtained in the reconstruction. In other words the support information will enable one to fill the spectral hole $D$. The radius $r_{0}$ of this disk should be chosen according to the noise level in the hologram and to the magnitude of the deviation of the experimental PSF from the theoretical complex exponential PSF. It is typically of the order of 30 pixels for a $512 \times 512$ array. Let $r(x, y)$ be the estimate of $f$ obtained in the reconstruction; this Fourier-domain constraint $|F|$ reads as

$$
|F| : \hat{f}(\mu, \nu) = \hat{r}(\mu, \nu) \text{ if } |\mu, \nu| \notin D_{0} : |\mu^2 + \nu^2| < r_{0}^2.
$$

(21)

Both constraints lead to projections on convex sets. Indeed, $|S|$ expresses that $f$ belongs to a closed linear manifold $E_{S}$ (which is of finite dimension if we consider $f$ to be sampled). And the constraint $|F|$ expresses that $f$ belongs to a closed manifold $E_{F}$. If we denote by $P_{S}$ and $P_{F}$ the respective projection operators, $f$ must satisfy

$$
f \in E_{0} = E_{S} \cap E_{F};
$$

(22)

i.e., $f$ must be a fixed point of the two projectors,

$$
P_{S}f = f, \text{ and } P_{F}f = f.
$$

(23)

It is readily shown [see Ref. 21, for instance] that this condition is equivalent to

$$
Pf = f, \quad P = P_{F}P_{S};
$$

(24)

i.e., $f$ is a fixed point of the composition of the two projection operators.

B. Existence, Uniqueness, and Stability

If the applied constraints are physically realistic, i.e., if one does not force erroneous frequencies and if the imposed support is not underestimated, then $E_{0} = E_{S} \cap E_{F}$ will not be an empty set and there will exist (at least) one $f \in E_{0}$. To ensure the existence of $f$, it
is possible, if necessary, to loosen the frequency constraint \(|F|\) outside disk \(D\) and replace it with a constraint \(|F'|\) that more finely takes into account the defects in the PSF. In particular, the deviation of the experimental PSF is essentially an oscillation of the magnitude visible on the experimental PSF in Ref. 11, whereas the phase of the experimental PSF is, within a good approximation, still a parabola.\(^{11,15}\) According to Ref. 22 (cf. Appendix), it is possible to choose the following constraint:

\[ |F'| : \phi(\tilde{f}) = \phi(f), \text{ if } \lvert \mu^2 + \nu^2 \rvert \geq \rho_0^2, \]

\[ \text{and } a|\tilde{f}| \leq |\tilde{f}| \leq b|\tilde{f}|; \quad (25) \]

\(\phi\) denotes the phase of the function, and \(a\) and \(b\) are two functions close to 1 for the high frequencies and slightly constraining in the low frequencies, such as

\[ a(\mu, \nu) = 1 - c_1 - \frac{c_2}{\lvert \mu^2 + \nu^2 \rvert}, \]

\[ b(\mu, \nu) = 1 + c_1 + \frac{c_2}{\lvert \mu^2 + \nu^2 \rvert}, \quad (26) \]

where \(c_1\) and \(c_2\) are two constants to be chosen according to the magnitude of the oscillations. \(|F'|\) can then be viewed as a binarized version of \(|F'|\).

Convexity stems from the fact that the phase is imposed while the magnitude is left free within a certain range, in the same way as signal restoration from the phase gives rise to convex constraints (whereas restoration from the magnitude does not, because a circle is not convex). This prominent difference, incidentally, corroborates the well-known idea that phase contains more information than magnitude. More precisely, for \(\lvert \mu^2 + \nu^2 \rvert \geq \rho_0^2\) the value \(\tilde{f}\) of each point \((\mu, \nu)\) is forced to be on a segment of the set \(C\) of complex numbers, and for \(\lvert \mu^2 + \nu^2 \rvert < \rho_0^2\) this value is forced to be inside the sphere \(\lvert \tilde{f} \rvert = \rho_0\). \(c_1\) and \(c_2\) must be chosen so that \(a(\mu, \nu) \leq 0\) if \(\lvert \mu^2 + \nu^2 \rvert < \rho_0^2\). And a segment and the inside of a sphere are clearly convex.

The nonuniqueness of the solution is an intrinsic drawback\(^{23}\) of the method of restoration by projections onto convex sets. In the case of conoscopic holography this disadvantage is not overly troublesome since the initialization point is not random but, on the contrary, the best available estimate of the shape. In addition, in the case of constraint \(|F|\) there is a proof of uniqueness: assuming that \(f\) has a bounded support \(|S|\), corresponding to an object on a dark background, its Fourier \(\tilde{f}\) is known to be analytical. Hence if the latter is known outside disk \(D\) (constraint \(|F|\) and not \(|F'|\)), then \(f\) is uniquely determined.

Although it has not appeared explicitly so far, the maximum lateral resolution sought is no more than that of the sensor (and the CCD sensor is a natural low-pass filter). The aim of the restoration presented here is to fill the central spectral hole, i.e., to perform a Fourier interpolation (without an extrapolation) by use of the support constraint. In these conditions the problem is well posed\(^{24}\); that is to say, the recovered shape varies continuously as a function of the hologram. Moreover, the amount of interpolation to perform \(\eta\), defined by

\[ \eta = \left[ \int \int 1_S(x, y)dx dy \int \int 1_P(\mu, \nu)\mu d\mu d\nu \right]^{1/2}, \quad (27) \]

where \(1_S\) and \(1_P\) are the characteristic functions of the support of \(f\) and of disk \(D\), respectively, gives an estimation of the condition number of the restoration. The restoration thus is all the better conditioned, as the object’s support and the spectral hole are smaller, which corresponds to both intuition and experience.

C. Restoration Algorithm

The chosen restoration algorithm consists of imposing constraints \(|S|\) in real space and \(|F|\) in Fourier space successively to a first estimate of \(\tilde{f}\), using fast Fourier transforms (FFT’s) to go from one domain to the other. This iterative method was invented by Gerchberg and Saxton\(^{26}\) to recover a complex amplitude from its magnitude in real and Fourier domains, and it was used by Gerchberg\(^{26}\) for superresolution purposes with, this time, a support constraint in the real domain. The convergence for this latter problem was derived by Papoulis.\(^{27}\) This method has had considerable success and in particular has been used for the recovery of a signal from its phase\(^{28}\) or from its magnitude.\(^{29,30}\)

This algorithm, which searches a fixed point \(f\) of \(P\), consists of two steps: for a given \(f^n\), the \(n\)th iteration estimate of \(f\),

\[ f_n = P_Sf_n, \quad (28) \]

is the function satisfying \(|S|\) that is closest to \(f^n\). Applying the frequency constraint \(|F|\) to \(f_n\) in practice, after an FFT yields the next estimate of \(f\) after an inverse FFT:

\[ f_{n+1} = P_Ff_n = P_FP_Sf_n = P_{n+1}f_0. \quad (29) \]

The initialization of these successive projections is, as mentioned previously, the output of the reconstruction algorithm. In order to improve the convergence properties of this method, one may think of relaxing the projection operators: let \(Q\) and \(I\) be a projection and the identity operators, respectively, and let \(\lambda\) be a real number called the relaxation parameter; the relaxed operator \(R\) is defined as

\[ R = I + \lambda(Q - I) = \lambda Q + (1 - \lambda)I. \quad (30) \]

Since \(R\) and \(Q\) have exactly the same fixed points, relaxing \(Q\) does not change the set of possible limits but modifies the speed of convergence (and possibly the convergence). It is worth noticing that this projection method corresponds to the minimization of a regularizing functional (defined in Subsection 4.2 of...
by the method of steepest descent\cite{31} with a fixed step in the case of a nonrelaxed projection.

D. Convergence

Under assumptions that we discussed above,

\( E_S, E_F \) are closed convex sets,

\[ E_0 = E_S \cap E_F \neq \emptyset, \]

\[ T_S = I + \lambda_S P_S - I, \quad T_F = I + \lambda_F P_F - I, \]

\[ T = T_F T_S, \quad (31) \]

then there are two important results\cite{20}:

*Theorem 1.* For any \( f_0 \), for \( 0 < \lambda_S < 2 \), and for \( 0 < \lambda_F < 2 \), \( T^n f_0 \) converges weakly to a point \( f^* \) of \( E_0 \).

*Corollary.* If, moreover, \( E_S \) and \( E_F \) are manifolds, \( T^n f_0 \) converges strongly toward \( P_0 f_0 \), where \( P_0 \) is the orthogonal projection operator onto \( E_0 \).

This theorem and its corollary apply to our problem constraints \( \{ S \} \) and \( \{ F \} \). For these constraints we even know that \( E_0 \) consists of a single point.

If the frequency constraint \( \{ F \} \) is replaced with \( \{ F' \} \), \( E_F \) is no longer a manifold; thus the corollary is no longer valid. But the interior of \( E_F \) is not empty by construction in particular, for well-chosen \( a \) and \( b \) functions, it contains \( E_F \), thus the following result (derived in Ref. 32 and cited in Ref. 21) can be used.

*Theorem 2.* If \( E_S \cap E_F \neq \emptyset \), then \( T^n f_0 \) of Theorem 1 converges strongly toward its weak limit \( f^* \), and the convergence rate is geometrical.

The optimization of the relaxation parameters is a crucial problem only if the number of iterations that is necessary to reach an acceptable solution is important. It can then be shown (a result by Levi cited in Ref. 33) that these parameters can be modified at each iteration to speed up convergence. In the case of conoscopic holography, convergence does not take more than a dozen iterations in practice, which make these successive optimizations unnecessary. Yet two results on this more general algorithm are worth noting.

If parameters \( \lambda_S \) and \( \lambda_F \) are optimized independently at each step (one iteration consisting of the two steps \( f_n \rightarrow f_n' \) and \( f_n' \rightarrow f_{n+1} \)), then the two optimal parameters satisfy

\[ \lambda_{S/F}^{opt} > 1, \quad (32) \]

and, in particular, if the range of one of the projection operators is a linear manifold, the corresponding relaxation parameter \( \lambda^{opt} \) satisfies

\[ \lambda^{opt} = 1. \quad (33) \]

Hence \( \lambda_S = 1 \) is optimal for the per step optimization.

Such an optimization may be underoptimal compared with the joint optimization of both parameters on a whole iteration, but if the relaxed projector onto the linear manifold is applied after the other one (i.e., \( T = T_S T_F \) instead of \( T_F T_S \)), then \( \lambda_S = 1 \) is also optimal for the per iteration optimization. Indeed, for any value of the relaxation parameter of the first constraint, if the second parameter is optimal per iteration, it is a fortiori also optimal per step. Moreover, since the initialization point \( f_0 \) is the output of the reconstruction algorithm, it satisfies constraint \( \{ F \} \) and \( \{ F' \} \); thus

\[ f_0 = T_F f_0, \quad (34) \]

and one can consider that \( \{ F \} \) is applied first and \( \{ S \} \) second. In conclusion, \( \lambda_S = 1 \) is the optimal parameter and corresponds to a nonrelaxed projection. Since the number of iterations required in practice is reasonably small, \( \lambda_F \) will also be taken equal to unity.

5. Experimental Results

The experimental setup, Fig. 2, consists of the following:

- A collimated 10-mW He–Ne laser, used for the

![Fig. 2. Experimental setup: L.C. light valve, liquid-crystal light valve.](image-url)
alignment of the system elements, the calibration (acquisition of the PSF) and the illumination of objects.

- Two translucent squares on a dark background (photographic plate) as the object, with a rotating ground-glass diffuser placed before it to eliminate speckle.
- A mask (a gray-level slide transferred onto a photographic plate) and a personal-computer-driven liquid-crystal light valve (Meadowlark LVR-0.7-CUS), mounted together on a rotation stage (Microcontrôle, also personal-computer driven).
- A 50-mm / 1.8 Nikkor lens having the mask in its front focal plane, to image the object into the system.
- A 50-mm-long calcite crystal (20-mm in diameter), an output circular polarizer, and a CCD camera (Cohu 4712), whose images are digitized on 512 × 512 pixels (Matrox PIP-1024 board).

The mask and the liquid-crystal light valve permit a modulation of the amplitude and of the input polarization of the incident light field, respectively, in order to obtain the complex PSF $R_e$ (see Ref. 11 for details). Owing to the need to eliminate laser speckle with a rotating diffuser, too much light would have been lost if the hologram had not been taken in two steps: the first part of the acquisition was done with the translucent square (on a photographic plate) at a given location and the diffuser behind it, then the plate and the diffuser were moved both laterally and longitudinally, and the acquisition went on. The future illumination system will obviously have to be spatially incoherent. The 3-D image of these two squares through the lens is approximately 0.9 mm in width and 1.2 mm in depth, corresponding to a variation of the number of fringes, $\Delta F$, of ±2.

The hologram is the sum of 100 snapshots and is shown in Fig. 3 (the summation of several snapshots is necessary to the acquisition of the imaginary part of the PSF, cf. Ref. 11, and improves the signal-to-

![Fig. 3](image1.png)

Fig. 3. (a) conoscopic hologram of the two squares, real part; (b) conoscopic hologram of the two squares, imaginary part.

![Fig. 4](image2.png)

Fig. 4. (a) Reconstruction of the image of the two squares (real part of the refocused hologram); (b) reconstruction of the edges of the two squares (imaginary part of the refocused hologram).
noise ratio of the hologram. The hologram was resampled to compensate for the fact that the CCD pixels were not square, which explains why the pictures are not square. The 2-D reconstruction (refocused hologram) is shown in Fig. 4. The visible difference in size of the two squares stems from the fact that the lens imaging the object into the system is not afocal, and consequently it has magnification that varies with the longitudinal position. It can be seen from the imaginary part that this reconstruction was performed into a plane between the two planes the object consists of, because the two lines corresponding to the edge of the object are bright, then dark (toward the outside of the square), for the closer plane, and dark, then bright, for the farther plane.

The output of the 3-D reconstruction algorithm (with parameter \( w \) equal to 9 pixels) is shown in Fig. 5. One can see the expected shape consisting of a black square and a white square, corresponding, respectively, to a plane behind and in front of the plane of the 2-D reconstruction (plane of refocusing). But this shape is embedded in a large, low-frequency artifact. The restoration algorithm presented above permits the recovery of the low spatial frequencies. To apply the support constraint \( S \), one forces to zero the value of pixels whose real part of the 2-D reconstruction (i.e., of the image) is less than 15% of the maximum intensity value. The frequency constraint \( F \) consists in forcing only the value of frequencies outside disk \( D \) to their value obtained in the reconstruction; the radius of this disk is \( \sim \)30 pixels for a 512 \( \times \) 512 array. The result of Fig. 6 is obtained after half a dozen iterations, the computing bulk of each iteration being essentially two FFT's.

6. Conclusion

I have presented an algorithm for the 3-D reconstruction of an opaque object from its conoscopic hologram. Yet the reconstruction is unstable for the low spatial frequencies, which are the ones that determine the overall shape, because the signal-to-noise ratio of the object’s shape decreases in these frequencies. Thus in order to restore them, I have proposed and studied the properties of an iterative method that takes advantage of knowledge of the object’s support (obtained as a by-product of the 3-D reconstruction) to make the reconstruction less sensitive to noise and to defects in the PSF. Finally, encouraging 3-D results have been presented that validate the potentialities of conoscopic holography as a 3-D imaging technique. The main problem that remains to be tackled, in order to achieve 3-D reconstructions of large objects, is the weakness of the shape signal (imaginary part of the refocused hologram), since it is typically of the same order of magnitude as the sidelobes of the intensity signal (real part of the refocused hologram). This future research will have to focus on enhancements and/or on a better modeling of the experimental PSF and possibly on the use of the experimental PSF itself in the 3-D reconstruction.

Appendix A

The deviation of the experimental PSF from the theoretical one is modeled by an amplitude modulation \( e(x, y) \). According to theorem 1 of chapter 8, paragraph 1, of Ref. 22, if the chirp function \( R \) is modulated by an envelope \( e(x, y) \), then the Fourier transform of the product \( e(x, y)R(x, y) \) is approximately \( e(\mu, \nu) \Re FT(R, \mu, \nu) \) (where \( \kappa \) is a constant); thus the deviation of the 2-D transfer function to the theoretical one is also an amplitude modulation without an alteration of the phase. By taking the derivative of this function with respect to \( z^2 \) one can easily show that the relative error on the shape \( Ix^2 - z^2 \) in Fourier space is still an amplitude modulation and is bounded by \( c_1 + c_2|\mu^2 + \nu^2| \), where \( c_1 \) and \( c_2 \) are constants, which justifies the form chosen for functions \( a \) and \( b \).

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References