ECE 299 Holography and Coherent Imaging

Lecture 3 Three exercises in hologram analysis

David J. Brady
Duke University

www.disp.duke.edu/~dbrady/courses/holography
Outline

1. Resolution of off axis holograms
2. Holographic magnification
3. Phase and amplitude holograms
Problem 1. Resolution of off axis holograms

A hologram is recorded of an object at a range of 10 cm using 532 nm light. The angle between the reference and signal beams is 20 degrees.

1. Estimate the maximum fringe frequency on the holographic plate
2. Estimate the angular resolution of the holographic image
3. Estimate the spatial resolution of the image at the object plane
4. How large should the holographic plate be?
5. What sampling period is required to simulate this system? How large an object can be simulated?
\[ \lambda = 532 \text{ nm} \]

\[ f(x, y) \]

\[ 10 \text{ cm} \]

\[ I = |A e^{i 2\pi \frac{x \sin \theta}{\lambda}} + f(x, y)|^2 \]

\[ = |A|^2 + |f|^2 + A e^{i 2\pi \frac{x \sin \theta}{\lambda}} f^*(x, y) + \text{c.c.} \]
\[ U_\text{max} = \frac{B + u_0}{2} \leq u_0 \]

\[ U_\text{max} \leq \frac{u_0}{3} + u_0 = \frac{4u_0}{3} = 857 \text{ lpm} \quad \gamma = 1.2 \text{ mm} \]

\[ U_0 = \frac{\sin \frac{\pi}{3}}{2} = 6.4 \cdot 10^5 \text{ m}^{-1} = 642 \text{ mm}^{-1} \]
Angular resolution

\[ \beta = \frac{2}{3} u_0 = 429 \text{ lp/mm} \]

\[ \delta x \approx \frac{1}{\beta} = 2.3 \mu \text{m} \]

\[ \delta \Theta = \frac{\delta x}{z} = 23 \text{ mrad} \approx 5 \text{ arc seconds} \]
Holographic plate size.

\[ I(x) = \left| R e^{i2\pi u_x} + \tilde{f}(x) \right|^2 \rho(x) \]

\[ = \rho(x) \left( |R|^2 + |f|^2 \right) + R e^{i2\pi u_x} \tilde{f}(x) \rho(x) + c.c. \]

\[ \Delta x = 0x + \frac{\Delta z}{\Delta x} \approx 22 \text{ mm} \]
Sampling period

\( \Delta x = 1.1 \text{ mm} \)

Assume \( n = 1024 \)

1 mm

Object

hologram
Problem 2. Magnification

1. Describe holographic magnification using a change of wavelengths
2. By what factor may one reasonably magnify an object at range $R$ using holography?
Holographic magnification

Boundary condition

\[ f(x, y) \]

Diffracted field

\[ \tilde{g}(x', y') = \iint f(x, y) e^{i\pi (ax'^2 + by'^2)} \, dx \, dy \]
Holographic magnification

\[ f = \mathcal{F} \left\{ \hat{f}(u,v) e^{i \pi \xi_1 (u^2 + \sigma^2)} \right\} \]

Object

Reconstruction at \( d_2 \)

Holographic image

Image field

\[ \mathcal{F}^{-1} \left\{ \hat{f}(u,v) e^{i \pi \lambda_1 z_1 (u^2 + \sigma^2)} e^{-i \pi \lambda_2 z_2 (u^2 + \sigma^2)} \right\} \]

\[ \lambda_1 z_1 = \lambda_2 z_2 \]
Object appears $\frac{d_2}{d_1}$ times closer
and is thus magnified in angular extent.
Holographic microscopy

Can one use holography to see subwavelength features?

Version 1 loss of information on propagation

\[ \frac{1}{\lambda} \]

maximum spatial frequency \( f \)
Holographic Microscopy

Can one use holography to observe sub-wavelength features?

\[ e^{i \frac{2\pi}{\lambda^2} (x-x')^2 + y^2} \]

\[ e^{i \frac{2\pi}{\lambda^2} (x-x')^2 + y^2} \]

Hologram must resolve frequency

\[ \Delta x > \frac{\lambda^2}{2\Delta \lambda} \]

\[ \Delta x \frac{\Delta \lambda}{\lambda^2} \]

Site of hologram must be \( 0 > \frac{\Delta \lambda}{\Delta x} \)

Frequency must be \( \leq \frac{1}{2\lambda^2} \)
Conservation of Space - Bandwidth product

Would re-sampling accurately describe field propagation?
Problem 3. Phase and amplitude holograms

1. What is the maximum diffraction efficiency for an amplitude hologram?
2. What are the relative diffraction efficiencies for higher orders of a phase hologram?
Amplitude hologram

- Commonly made using silver halide emulsions
- Transmission \( \Phi(x, y) \) is real

\[ \Phi \leq \Phi(x, y) \leq 1 \]
Example amplitude hologram

\[ A e^{i \pi u x} e^{i 2 \pi \frac{1}{x^2} - u_0^2 x} \]

\[ f_0 e^{i 2 \pi t} \]

\[ f(x) = |A|^2 + 1 f_0^2 + |A|/f_0 \cos (2 \pi u_0 x + \phi) \]

Max modulation depth for \( f_0 = A \)

\[ f = \frac{1}{2} \left( 1 + \cos (2 \pi u_0 x) \right) \]
Amplitude diffraction efficiency

\[ \frac{1}{4} e^{i \pi n_0 x} \]

\[ \frac{1}{2} \left( 1 + \cos(2\pi n_0 x) \right) \]

\[ \frac{1}{4} e^{i \pi n_0 x} \]

Compute diffraction

\[ h(x) = \text{rect} \left[ \cos \left( \pi n_0 x \right) \right] = \sum_{n=-\infty}^{\infty} \frac{\sin \pi n}{\pi n} e^{i \pi n x} \]

\[ t(x) \]
Phase hologram

recording modulates index of refraction $n$

$$\Delta n = |r|^2 + |f|^2 + rf^* + r^*f$$

$$\delta \phi = 2\pi \frac{d}{l} \Delta n$$

$$f(x, y) = e^{i\phi(x, y)}$$
Example

$$D \psi = \alpha_0 \sin(2\pi u_0 x)$$

$$\hat{f}(x,y) = e^{i 2\pi D \psi \frac{1}{\lambda}} \sin(2\pi u_0 x)$$

$$= \sum_{q=-\infty}^{\infty} J_q \left( \frac{2\pi \alpha_0 \rho}{\lambda} \right) e^{i 2\pi u_0 q x}$$

*Jacobi - Anger expansion*
\( Bessel J[0,1] = .76 \)

\( 11 = .44 \)

\( 21 = .11 \)

\( 31 = .019 \)