Single disperser design for compressive, single-snapshot spectral imaging

Ashwin A. Wagadarikar, Renu John, Rebecca Willett and David J. Brady*

Department of Electrical and Computer Engineering, Duke University, Durham, NC, 27708

ABSTRACT

Recent theoretical work in “compressed sensing” can be exploited to guide the design of accurate, single-snapshot, static, high-throughput spectral imaging systems. A spectral imager provides a three-dimensional data cube in which the spatial information of the image is complemented by spectral information about each spatial location. In this paper, compressive, single-snapshot spectral imaging is accomplished via a novel static design consisting of a coded input aperture, a single dispersive element and a detector. The proposed “single disperser” design described here mixes spatial and spectral information on the detector by measuring coded projections of the spectral datacube that are induced by the coded input aperture. The single disperser uses fewer optical elements and requires simpler optical alignment than our dual disperser design.1 We discuss the prototype instrument, the reconstruction algorithm used to generate accurate estimates of the spectral datacubes, and associated experimental results.

Keywords: Compressed sensing, Spectral imaging, Coded aperture

1. INTRODUCTION

Spectral imaging is a useful tool for many applications because of the additional dimension of information it provides about the scene being imaged. Conventional imagers produce two-dimensional images that represent the spatial information in the scene with a scalar value attached to each pixel location. In contrast, a spectral imager produces a two-dimensional spatial array of vectors which represents the spectrum at each pixel location. The resulting three-dimensional dataset containing the two spatial dimensions and one spectral dimension is known as the spectral data cube. Applications requiring a spatial map of spectral variation include environmental remote sensing,2 military target discrimination,3 astrophysics,4 biochemistry5 and biomedical optics.6,

The simplest spectral imagers combine a pushbroom (linear scanning) or tomographic (rotational scanning) front-end with a slit-based dispersive spectrometer. Recent designs have focused on direct-view methods that maximize the light gathering efficiency.1,7-9 Two notable designs include the computed tomography imaging spectrometer (CTIS)9 and the dual disperser imaging spectrometer (DDIS),1 as both instruments capture a scene’s spatial and spectral content without any type of scanning and do so in a single snapshot. The CTIS instrument avoids scanning by utilizing a computer generated hologram that splits the scene into a zeroth-order undispersed image and multiple spectrally-dispersed images that are captured by a detector. The intensity data from the detector snapshot is then processed using computed tomography algorithms to reconstruct the spectral datacube. The DDIS system design consists of two sequential dispersive arms, each with a 4-f geometry, commonly used as a traditional dispersive spectrometer. The two arms are arranged in opposition so that the second arm exactly cancels the dispersion introduced by the first arm. A binary-valued coded aperture is placed in the plane separating the two arms. Such a design applies spatially-varying, spectral filter functions with narrow features. Through these filters, the CCD measures a projective measurement of the input scene in the spectral domain. Recovery of the spectral datacube is performed using reconstruction techniques based on compressed sensing frameworks.10

In this paper, compressive, single-snapshot spectral imaging is accomplished via a novel static design dubbed the “single disperser imaging spectrometer” (SDIS) that consists of a coded input aperture, a single dispersive element and a detector. Like the DDIS, the SDIS does not directly measure each voxel in the desired three-dimensional spectral datacube. Instead, it collects a small number (relative to the size of the data cube) of coded measurements and a sparse reconstruction method is used to estimate the spectral image from the noisy projections. A DDIS instrument cannot reconstruct the spectrum of a point source object. In contrast, the SDIS can reconstruct the spectrum of a point source, provided that the

*Send correspondence to David J. Brady: dbrady@duke.edu
source spatially maps to an open element on the coded input aperture. Furthermore, the SDIS design benefits by requiring a fewer number of optical elements, making optical alignment much easier. However, the proposed design mixes spatial and spectral information at the detector. Thus, unlike in the DDIS design, a raw measurement of a scene on the detector rarely reveals spatial structure of the scene.

Since spectral imaging uses a two-dimensional focal plane to characterize three-dimensional data cubes, sampling is generally not of full rank in the space, time or spectral degrees of freedom. Pushbroom systems measure the full data cube but require temporally static scenes. CTIS systems miss a cone in Fourier space. Our DDIS and SDIS designs draw on recent progress in generalized and compressive sampling theory in an attempt to use natural sparsity to enable full data cube characterization from low rank measurements.

In the following sections, we detail our system design and demonstrate the practical application of the SDIS to spectral imaging through discussion of a prototype instrument and the reconstruction algorithm used to generate accurate spectral datacubes from an under-determined set of noisy projections.

2. SYSTEM MODEL

A schematic of the static, single-snapshot coded aperture spectral imager is shown in Fig. 1. A standard imaging lens is used to form an image of a remote scene in the plane of the coded aperture. The coded aperture modulates the spatial information over all wavelengths in the spectral cube with the coded pattern. Imaging the cube from this plane through the dispersive element results in multiple images formed at wavelength-dependent locations in the plane of the detector array. The spatial intensity pattern in this plane contains a coded mixture of spatial and spectral information about the scene.

![Figure 1. Schematic of the Single Disperser Imaging Spectrometer (SDIS).](image)

The spectral density entering the instrument and being relayed to the plane of the coded aperture can be represented as \( f_0(x, y; \lambda) \). If we represent the transmission function printed on the coded aperture as \( T(x, y) \), the spectral density just after the coded aperture is:

\[
f_1(x, y; \lambda) = f_0(x, y; \lambda) T(x, y),
\]

After propagation through the coded aperture, the imaging optics and the dispersive element, the spectral density at the
detector measurement in operator form can be written as:
\[
f_2(x, y; \lambda) = \iint \delta(x' - [x + \alpha(\lambda - \lambda_c)]) \delta(y' - y) f_1(x', y'; \lambda) \, dx' \, dy'
\]
\[
= \iint \delta(x' - [x + \alpha(\lambda - \lambda_c)]) \delta(y' - y) f_0(x', y'; \lambda) T(x', y') \, dx' \, dy'
\]
\[
= f_0(x + \alpha(\lambda - \lambda_c), y; \lambda) T(x + \alpha(\lambda - \lambda_c), y),
\]
where the Dirac delta functions describe propagation through unity-magnification imaging optics and a dispersive element with linear dispersion \(\alpha\) and center wavelength \(\lambda_c\). The detector array is insensitive to wavelength and measures the intensity of incident light rather than the spectral density. Thus, the continuous image on the detector array can be represented as:
\[
g(x, y) = \int f_0(x + \alpha(\lambda - \lambda_c), y; \lambda) T(x + \alpha(\lambda - \lambda_c), y) \, d\lambda.
\]

Since the detector array is spatially pixelated with pixel size \(\Delta\), \(g(x, y)\) is sampled across both dimensions on the detector. In the presence of noise, \(w\), the detector measurements can be represented as:
\[
g_{nm} = \iint g(x, y) \text{rect} \left( \frac{x}{\Delta} - m, \frac{y}{\Delta} - n \right) \, dx \, dy + w_{nm}
\]
\[
= \iiint f_0(x + \alpha(\lambda - \lambda_c), y; \lambda) T(x + \alpha(\lambda - \lambda_c), y) \text{rect} \left( \frac{x}{\Delta} - m, \frac{y}{\Delta} - n \right) \, dx \, dy \, d\lambda + w_{nm},
\]

Reconstructions of spectral datacubes from the physical system benefit if the coded aperture feature size is an integer multiple, \(q\), of the size of the detector pixels, \(\Delta\). This avoids the need for sub-pixel positioning accuracy of the coded aperture. The aperture pattern \(T(x, y)\) can be represented as a spatial array of square pinholes, with each pinhole having a side length \(q\Delta\) and \(t_{n'n''m'}\) representing an open or closed pinhole at position \((n', m')\) in the pinhole array.
\[
T(x, y) = \sum_{m', n'} t_{n'm'} \text{rect} \left( \frac{x}{q\Delta} - m', \frac{y}{q\Delta} - n' \right).
\]

With this representation for the aperture pattern, the detector measurements as represented in Eq. 4 become:
\[
g_{nm} = \sum_{m'n'm'} t_{n'm'} \iiint \text{rect} \left( \frac{x + \alpha(\lambda - \lambda_c)}{q\Delta} - m', \frac{y}{q\Delta} - n' \right) \text{rect} \left( \frac{x}{\Delta} - m, \frac{y}{\Delta} - n \right) \times \]
\[
f_0(x + \alpha(\lambda - \lambda_c), y; \lambda) \, dx \, dy \, d\lambda + w_{nm}.
\]

Denoting the source spectral density \(f_0(x, y; \lambda)\) in operator form as \(f_{ijk}\) and the aperture code pattern \(T(x, y)\) as \(t_{ij}\), the detector measurement in operator form can be written as:
\[
g_{nm} = \sum_k \int f_{(m+k)n+k} t_{(m+k)n} + w_{nm}
\]
\[
= \mathbf{H} f + w_{nm}
\]
where \(\mathbf{H}\) is a linear operator that represents the system forward model.

### 3. Reconstruction Method

In this section, we describe the reconstruction method used to get the spatial-spectral information of the scene from the detector measurement. The compressed sensing methodology implemented by the SDIS builds upon a crucial assumption made about the spectral datacubes to be reconstructed, namely that the sources in the scene have piecewise smooth structure, both spatially and spectrally.
The spectral datacube can be represented as:

\[ f = W \theta, \]

where \( W \) is the inverse wavelet transform and \( \theta \) represents the three-dimensional wavelet coefficients of the spectral datacube \( f \). The reconstruction methodology used to estimate the spectral datacube from an SDIS detector measurement assumes that the datacube has a sparse representation in the wavelet basis; i.e. that \( \theta \) contains mostly zeros and a relatively small number of large coefficients. The detector measurement can be represented as:

\[ g_{nm} = H W \theta + w_{nm}, \]

where \( H \) is a representation of the system forward model described in Section 2, \( W \) is the inverse wavelet transform, \( \theta \) are the wavelet coefficients of the spectral datacube, and \( w_{nm} \) is noise.

If the spectral datacube, \( f \), consists of \( \{n \times n\} \) spatial channels with \( m \) spectral channels, it can be represented as a cube of size \( \{n \times n \times m\} \). The corresponding detector measurement, \( g_{nm} \), can be represented as a matrix of size \( \{n \times (n + m - 1)\} \). If we represent \( f \) and \( g_{nm} \) as column vectors, the linear operator matrix, \( H \), can be represented as a matrix of size \( \{n(n + m - 1) \times (n^2m)\} \). The cube of wavelet coefficients of the spectral datacube, \( \theta \), in vector form is of size \( \{n(3n\log_2(n) + n)m \times 1\} \). The size of this cube reflects the fact that the wavelet decomposition of the spectral datacube is performed as a two-dimensional undecimated transform on each of the ‘m’ spectral bands. The undecimated transform is used to ensure that the resulting method is translation invariant.

An estimate, \( \hat{f} \), for the spectral datacube can be found by solving the problem:

\[ \hat{f} = W \left[ \arg \min_{\theta'} \| g_{nm} - H W \theta' \|_2^2 + \tau \| \theta' \|_1 \right]. \]

The first term in this optimization equation minimizes the \( \ell_2 \) error between the measurements modeled from the estimate and the true measurement. The second term is a penalty term that controls the amount of smoothness of the estimate. In this formulation, \( \tau \) is the tuning parameter for the penalty term and higher values of \( \tau \) yield sparser estimates of \( \theta \). The solution of this nonlinear optimization problem has received significant attention recently.\textsuperscript{11–13} To reconstruct spectral images, we use the Gradient Projection for Sparse Reconstruction (GPSR) method developed by Figueiredo et al.\textsuperscript{14} This approach is based upon a variant of the Barzilai-Borwein gradient projection method,\textsuperscript{15} and has code available online at http://www.lx.it.pt/~mtf/GPSR/. This approach is fast and has performed well in our experiments.

### 4. SIMULATION RESULTS

To test the SDIS concept, a Matlab simulation was conducted using a phantom spectral datacube. For the purpose of the simulation, an approach that would result in easy visualization of the results of the reconstruction algorithm was desired. The phantom datacube was generated by converting a 256 \( \times \) 256 RGB color image of peppers, as shown in Fig. 2(a), to a synthesized datacube with a 15 channel spectrum defined for each pixel in the image.

The spectrum at each pixel corresponded to a weighted sum of RGB filter functions. Using the synthesized datacube, the detector measurement was simulated by passing the datacube through the system forward model. Figure 2(b) shows the aperture code used in the measurement process. The feature sizes on the aperture code were assumed to be the same as the size of the detector pixels.

The resulting 256 \( \times \) 270 detector measurement is shown in Fig. 2(c). Note that the simulation uses the ideal aperture code pattern and does not account for alignment and assembly issues that are encountered in the experimental measurement process.

Using the aperture pattern and the simulated detector measurement, the GPSR reconstruction algorithm was used to estimate the spectral datacube. The algorithm was run on an AMD Athlon 64 dual 3.8 GHz processor for 100 iterations and required 138 minutes. The resulting spectral datacube estimate was converted to a two dimensional image of RGB vectors using the RGB filter functions. The RGB image generated from the estimated spectral datacube is shown in Fig. 2(d). Although there are some artifacts present, this simulation demonstrates that the reconstruction algorithm has significant potential to generate datacube estimates of complex spatio-spectral scenes from detector measurements.
Figure 2. (a) 256 × 256 RGB image used to generate the spectral datacube. (b) 256 × 256 code pattern used for reconstruction of the simulated spectral datacube. The feature size was the same as the size of the detector pixels. (c) 256 × 270 simulated detector measurement of phantom spectral datacube. (d) 256 × 256 RGB image generated from the estimated spectral datacube.
5. EXPERIMENTAL RESULTS

To experimentally verify the SDIS spectral imaging idea, we constructed an SDIS prototype as shown in Fig. 1. The prototype consists of three lenses having a $\text{f/}$# of 1.4 and a focal length of 22.5 mm (Schneider Optics Inc.), an equilateral prism (part C43-495, Edmund Optics) as a dispersive element and a Coolsnap monochrome charge-coupled device (CCD) detector with $1040 \times 1392$ pixels that are $4.65 \mu m$ square each. The aperture code is an order 192 S-matrix code\textsuperscript{16} with mask features that are 4 CCD pixels wide and 4 CCD pixels tall, with 2 CCD pixel tall completely closed “dead rows” added between the code rows.

Given the system geometry and the low linear dispersion of the equilateral prism, the number of CCD columns illuminated when white light was allowed to pass through the system was less than half the width of the CCD array. Thus, the spectral range of the instrument is limited by the quantum efficiency of the CCD image sensor.

To generate an estimate of the spectral datacube representing the scene, the reconstruction algorithm requires two inputs - the aperture code pattern and the two dimensional array of measurements from the detector. Instead of using the code pattern printed on the aperture code as the code pattern used for the reconstruction, the instrument is uniformly illuminated with a single wavelength source in the form of a $543 nm$ laser. This generates a single image of the entire aperture code pattern after it has propagated through the optics. Figure 3(a) shows such a code pattern. This pattern is actually downsampled by a factor of two in the row and column dimensions for reasons that will be explained later in this section. While the mask features in the center of the field are clear and crisp, optical distortions play a major role and are easily noticeable at the edges of the field in the image. These distortions blur the mask features and make accurate reconstructions of the spectral datacubes more difficult for the reconstruction algorithm.

Figure 3(b) demonstrates a downsampled CCD measurement of a scene consisting of a ping pong ball illuminated with a $543 nm$ green laser and a $632 nm$ red laser. Given the low linear dispersion of the prism, there is spatial-spectral overlap of the aperture code-modulated images of the ball generated by each wavelength.

![Figure 3](image)

(a) Code pattern used for reconstruction of the spectral datacube. The distortions at the edge of the field are detrimental to accurate reconstruction of the spectral datacubes. (b) Detector measurement of a ping pong ball illuminated with a $543 nm$ green laser and a $632 nm$ red laser.

The downsampling of the code pattern and the CCD measurement helps reduce the time needed by the reconstruction algorithm to generate an estimate of the spectral datacube. Reconstruction of the spatial features of the ping pong ball over a spectral range of $535 nm$ to $640 nm$ took 55 iterations and lasted 1100 seconds or about 18 minutes. The GPSR method was initialized with $\tau = 0.1$.

Feeding the CCD measurement and the code pattern to the reconstruction algorithm results in the generation of an estimate for the spectral datacube. Figure 4 shows the spatial content of each of 30 wavelength channels between $535 nm$ to $640 nm$. We note that the mask modulation on the spatial structure visible in Fig. 3(b) has been removed in all the wavelength channels. Figure 5 plots the sum of the values in each wavelength channel.
Figure 4. Spatial content of each wavelength channel between 535 nm and 640 nm. Channel 3 corresponds to 543 nm and channel 28 corresponds to 632 nm.

Figure 5. Sum of content in each wavelength channel between 535 nm and 640 nm.

Ideally, all the spatial content would be limited to two wavelength channels and the power in each of these channels would be proportional to the power outputs of each laser. However, Fig. 4 shows that considerable spatial content is allocated to an additional wavelength channel next to the 632 nm channel. This is because the reconstruction algorithm assumed that the system response when the aperture code was fully illuminated with any wavelength was identical to the system response at 543 nm. Thus, it did not account for the anamorphic horizontal stretch of the image that is wavelength dependent. For a more accurate reconstruction, the reconstruction algorithm should be provided with a calibration datacube.
that consists of the aperture code pattern at all the wavelengths in the spectral range of the instrument.

6. CONCLUSION & FUTURE WORK

We have demonstrated the ability of a single-snapshot, static single disperser spectral imager (SDIS) to generate a spectral datacube from a set of measurements that code both the spatial and spectral information in the scene. Experimental verification of this ability is provided by demonstrating the reconstruction of a scene consisting of a ping pong ball illuminated with a 543 nm green laser and a 632 nm red laser. A number of issues are to be addressed through future work as part of the characterization of the SDIS instrument. First, the code pattern used for the reconstruction demonstrated in the previous section was not optimized. This could potentially be addressed by finding the code pattern that has a maximum auto-correlation peak or maximizes the incoherence of the matrix. Secondly, a new optical system is required that minimizes the optical distortions and thus increases the field of view of the instrument. Finally, experimental verification of the SDIS’ ability to generate an accurate datacube of more realistic scenes containing piecewise smooth spatial and spectral features is necessary. An example of such a scene would be group of colorful vegetables whose spectra span a broad range of the visible spectrum, as demonstrated by the simulation result presented earlier.

ACKNOWLEDGMENTS

This work was supported by the Defense Advanced Projects Agency Microsystems Technology Office through a collaborative project with Rice University ONR grant N00014-06-1-0610.

REFERENCES
